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A reputation management mechanism that incorporates accountability in online ratings

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Abstract Online reputation has a strong impact on the success of a seller in an e-marketplace. Also, buyers use it to choose an appropriate seller among a set of alternatives. The standard practice of determining the reputation of a seller is the aggregation of the feedbacks or the ratings reported by its buyers. Such a model of reputation formulation is vulnerable to misleading and unfair feedbacks. A seller may collude with a set of buyers to report good feedbacks while the quality of its product is poor. Also the buyers can report unfair feedbacks being irrational, malicious or competitors. A robust reputation management mechanism is the one which can not be manipulated by these unfair feedbacks. The existing reputation management models are either reactive or proactive. The reactive solutions intend to identify the unfair feedbacks and the proactive solutions propose incentive to the buyers to encourage them to report fair feedbacks. In this paper, we propose an incentive system that encourages the buyers to report fair feedbacks. We associate a buyer's reputation with a seller's reputation if the buyer has expressed its feedback about the seller. If the reputation of the seller decreases then the reputation of all buyers who had endorsed it (provided positive feedbacks) also decreases and vice versa. This means a buyer risks its own reputation by providing the feedback about a seller. In this paper, we show that such a mechanism is incentive compatible, i.e., it encourages the buyers to provide fair feedbacks. Using analytical and experimental analysis, we show the correctness of this reputation management system.

Keywords Reputation · e-marketplace · Trust

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1 Introduction

Online reputation has a strong impact on the success of a seller in an e-marketplace. Gretzel and Yoo [10] and Ye et al. [22] showed that the online reputation of the sellers impacts their sales. Also, a buyer may use it to choose the appropriate seller among a set of alternatives. Malik and Bouguettaya [14] showed that a reliable reputation system increases the buyer's trust on the e-marketplace [7]. Houser and Wooders [11] and Resnick and Zeckhauser [17] showed that the commercial success of e-marketplace such as eBay is attributed by its reputation management mechanism.

The standard practice of determining the reputation of a seller is the aggregation of the feedbacks or the ratings provided by its buyers. Such a model of reputation formulation is vulnerable to misleading or unfair feedbacks. A seller may collude with a set of buyers to provide good feedback while the quality of its product is poor. Also the buyers can provide unfair feedbacks being irrational, malicious or competitors. There are two types of unfair feedbacks:

- *False positive feedback* In this type of feedback, the buyer provides good feedback about a seller although the quality of the seller's product is poor. Such a behaviour of the buyer includes the scenarios when the seller colludes with a set of buyers and also, the cases where the buyers are irrational.
- *False negative feedback* In this type of feedback, the buyer provides bad feedback about a seller although the quality of the seller's product is good. Such a behaviour of the buyer may indicate that it is irrational or malicious.

The robust reputation management problem is concerned with developing certain mechanism so that it can not be manipulated by unfair feedbacks. The existing solutions to this problem are either reactive or proactive. The reactive solutions developed methods to identify the unfair feedbacks. Chen and Singh [3], Das and Islam [5], Dellarocas [6], Despotovic and Aberer [8], Kamvar et al. [13] and Yu and Singh [23], are the examples of the reactive solutions. A proactive solution determines incentive for the buyers to encourage them to report fair feedbacks. Often, this mechanism provides a payment to buyers for providing feedbacks and the mechanisms are designed in such a way that if the buyer provides fair feedback then it will get better payment. Examples of such incentives are [1, 2, 9, 12, 16, 21, 24].

The present literature of incentive design for reputation management model lacks a model that introduces 'risk' in providing unfair feedback. In this paper, we propose an incentive system that encourages the buyers to report fair feedbacks about the sellers as otherwise, their own reputation diminishes. The proposed reputation management model is motivated by the scenario where a brand decides to get associated a person for its advertisement. For example, a shoe company may endorse an athlete. Such an endorsement decision is based on the public image of the athlete. If the athlete is accused of any misbehaviour such as the usage of banned performance enhancement substances then, the shoe company would retract its endorsement as it does not want to be associated with a person who is accused of wrong practices. We use a similar mechanism. Our idea is to associate a buyer's reputation with a seller's reputation if the buyer expresses its feedback about the seller. If the reputation of the seller decreases then, the reputation of all buyers who had endorsed it (provided positive feedbacks) also decreases and vice versa. This means the buyer risks its own reputation by providing the feedback about the sellers.

Our reputation management mechanism is analogous to a share market. In a share market, a buyer wants to buy shares of the companies who are reputed and whose products are in demand. Also a buyer would like get rid of shares of companies whose performances (i.e., sales) are poor. In our reputation management model, the good feedbacks are represented as the events when a buyer agrees to buy a share and the bad feedbacks are the events when the buyer refuses to buy a share. Each seller can issue a set of shares. The share price indicates the reputation of the seller. A buyer can buy a share in exchange of its own reputation. The reputation of the buyer is estimated from the worth of its investments, i.e., the current value the shares it has bought. In this settings, we show the following:

- The buyers who are providing false good feedbacks becomes bankrupt, i.e., we show that the worth of their investments diminishes.
- The buyers who are providing false bad feedbacks becomes irrelevant. Note that, shares indicate good feedback. A buyer expresses bad feedback by not buying a share. We impose a restriction based on the reputation of the buyers that decides whether a buyer is allowed to buy a share or not. We show that, using such restriction we can isolate the buyers who had provided false bad feedbacks.
- Besides the above results, we show that the reputation of the good sellers increases w.r.t the reputation of the bad sellers.

1.1 Related work

As mentioned in [20], there are two types of mechanisms to identify unfair feedbacks. The endogenous mechanisms [3, 6] only use the feedback to determine an unfair feedback. These mechanisms are based on statistical properties of the feedbacks. Often these mechanisms use the history of feedbacks and assume that majority of feedbacks are fair. The exogenous mechanisms incorporate external information to determine whether a feedback is fair or unfair. Examples of such information includes the credibility of the buyers. Das and Islam [5] uses personalized similarity measure to rate the recommendation credibility. In this mechanism, the credibility of an evaluator is determined by the its peers who have interacted with it. Similar approach to determine the credibility is used in [8]. Kamvar et al. [13] uses the service trust as the parameter to determine the feedback credibility. But this mechanism is vulnerable in the situations where the service provider faces competition and may send unfair feedback about its competitors. Yu and Singh [23] proposes the weighted majority algorithm (WMA) that assigns weights in such a way that the relative weight assigned to the successful advisors is increased and the relative weight assigned to the unsuccessful advisors is decreased. Dellarocas [6] identifies the nearest neighbors of a buyer agent based on their preference similarities. Preference similarity is calculated using the number of their similar ratings for commonly rated sellers. After identifying the nearest neighbours of the buyer agent, cluster filtering is used to identify unfair rating. Whitby et al. [19] extends the reputation management systems developed in [4] to filter out unfair ratings using the iterated filtering. Chen and Singh [3] has used the buyers reputations in the calculation of the sellers reputation. Teacy et al. [18] proposed the TRAVOS model, which is a trust and reputation model for agent-based virtual organizations. This mechanism first estimates the accuracy of the current reputation advice based on the amount of accurate and inaccurate previous advice which is similar to the current reputation advice. Next, it adjusts reputation advice according to its accuracy. The aim of this task is to reduce the effect of inaccurate advice. However, this model assumes that seller agents act consistently, which might not be true in many cases.

There are several algorithms for designing incentives for reputation management system. Zhao et al. [24] has modelled the incentive system using a payment game in such a way that the agents who provide truthful feedbacks get more utility. Jurca and Faltings [12] proposed a payment scheme for feedback submission that encourages truthful feedback. In this mechanism, an agent gets paid if its feedback about a target agent matches the next feedback about the same target agent. The incentive model proposed in [9] is based on prisoners dilemma. In this model agents with truthful feedbacks gets better utility. Papaioannou and Stamoulis [16] proposed a sanctioning mechanism to obtain truthful feedback. In this model, in every transaction both parties submit a report about each other. If the reports in each transactions are not consistent then both parties are punished. Witkowski [21] studied the feasibility of payment system for eliciting truthful feedbacks for online auction systems. Ayday and Fekri [1] introduces an iterative probabilistic method for reputation management.

It should be noted that incentive schemes require an history of feedbacks and verified truthful feedbacks. This creates the problem of maintaining history of feedbacks and also it is vulnerable to collusion. In a contrast, our model of reputation management does not depend on such information. Few of the existing incentive mechanisms propose a payment for truthful feedbacks, but our reputation management model does not require any such payment.

1.2 Organization

This paper is organized in the following sections: in Sect. 2, we discuss a model of share market from a start-up prospective and informally discuss the analogous reputation management system. In Sect. 3, we introduce the formal model of the reputation management system. In Sect. 4 we provide analytical results. In Sect. 5, we present experimental analysis and we conclude the paper in Sect. 6.

2 A model of reputation management

The Reputation Management Mechanism (RMM) presented in this paper is as follows: the buyers (who provide feedbacks) place bet on the sellers reputation. If they loose (i.e., the seller's reputation diminishes and it had reported a good feedback about this seller) then they also loose their own reputation and vice versa. This mechanism introduces a risk in providing feedbacks as the buyers must choose good seller (whose reputation is expected to increase) to provide good feedbacks. There are two scenarios which illustrate this mechanism.

1. *Stock exchange* In a stock exchange, people can buy the shares of a firm. If the firm performs well, i.e., the demand for its product increases or the sale of its product increases then demand for its share also increases. Its shareholders can benefit if its share price increases and also, they will lose money if its share price decreases. Hence, objective of the share buyers is to buy the shares from the firms who are performing well or they believe that it will perform well in the near future.
2. *Endorsement* A firm may endorse a sports person. If the public image of the sports person diminishes then, the firm who is endorsing him/her may also lose reputation. Hence a firm chooses the sportsperson with a history of good public image.

In both scenarios, the utility (in stock exchange it's the value of investment and in endorsement it's the goodwill of the firm) of the first party (the stock buyer or the firm who is endorsing the players) depends on the performance of another party. Also, in both cases, the first party risks its utility based on its expectation about another party's performance.

Note that, in both scenarios, the first party, may prevent loss of its utility by disassociating itself from the second party. For example, a share holder may quickly sell the shares of a poorly performing firm. And, a firm may stop endorsing a player who found guilty of using illegal substances. In the context of RMM, such disassociation may be treated as a mechanism where a buyer withdraws its feedback about a seller. This mechanism will be a helpful mechanism if the sellers behaviour is not consistent. In this paper, we will assume that the sellers behaviour is consistent, i.e., they either sell good products or bad products. We intend to overcome this assumption in a future paper, where we intend to extend the RMM presented in this paper in the following way:

1. Sellers may not be consistent.
2. Buyers may withdraw their feedback.

The RMM imitates a share market as follows:

- *Currency* In our RMM, reputation will be treated as the currency. A new seller or a new buyer will be given a predefined amount of reputation once it enters the market.
- *Sellers* Initially each seller issues a set of shares. The number of such shares are uniform for all sellers. A share can be bought in exchange of reputation. The share price is determined by the demand of the share. In each interaction with a buyer, the seller provides the buyer the option to buy its share or refuse to buy its share. The buyer can buy a share by giving the seller a fraction of its own reputation (determined by the share price). The share price decreases if the buyer

does not buy the share and it increases as the buyer buys the share. The reputation of the seller at any time is its share price.

- *Buyers* The buyers receive an initial reputation from the reputation management systems. In each interaction with the sellers, it can buy its share if it gets the option to buy it. If it decides to buy then a fraction of its own reputation, equal to the share price of the share it wants to buy is transferred to the seller and the buyer gets a share. If it does not want to buy the share, which represents a negative feedback about the seller, the reputation management system stores this information that it has refused to buy the share. The reputation of the buyer depends on the share prices of the shares it has bought so far and the same for the shares it has refused so far. Its reputation goes up if the share prices of its investment goes up and vice versa. Also, its reputation goes down if the share prices of the shares it has refused go up and vice versa. This means, if a buyer has provided a positive feedback about a seller and other buyers have done the same then the buyer's reputation gets better. On the contrary, if a buyer has provided a negative feedback about a seller and other buyers do not agree with that, then its reputation gets worse.

In the next section we will present the formal model of the RMM. We will present a geometric model of association among the buyer's reputation and the seller's reputation.

3 Formal model

Let, there are m sellers $Sel = \{Sel_1, \ldots, Sel_m\}$ and $n + \epsilon$ buyers $Buy = \{Buy_1, \ldots, Buy_{n+\epsilon}\}$. As we assume that, the seller's behaviour is consistent, we allow one feedback from each buyer about each seller. Note that we want to promote the behaviour of the rational buyers (who provides correct feedbacks) and as the seller's behaviour is consistent, a rational buyer only need one chance to evaluate the seller's product.

Our objective is to develop a formal model of correlation between the reputation of the sellers and the reputation of the buyers. Such a model of correlation should achieve the following:

1. If the reputation of a seller increases then the reputation of all sellers who have provided positive reviews about the seller should also increase and vice versa.
2. If the reputation of a seller increases then the reputation of all sellers who have provided negative reviews about the seller should decrease and vice versa.

Now, with above objectives, we propose a model of reputation of the sellers. We use a circle of fixed diameter and each seller is assigned two points on the circumference of the circle. The acute angle on the center of circle from these two points gives the reputation of the seller. In fact, we use tan() of the quarter of this angle. If the reputation of the seller increases then such angle also increases and vice versa.

Definition 1 (*Share market and sellers*) The share market will be represented as a circle $\mathbb{S}$ (referred as market circle and shown in Fig. 1) such that,

- The center of $\mathbb{S}$ is the point s and its radius is r_s.
- Each seller $Sel_i \in Sel$ is assigned two points on the circumference of $\mathbb{S}$, denoted by $P(Sel_i)$. For example as shown in Fig. 1 the seller Sel_z is assigned two points are a and b. We refer these points as the seller points.
- $\angle(a, s, b)$ be the angle from a and b to s. Note that $\angle a, s, b \in [0, 180]$. We refer this angle as the seller angle.
- The share price for the seller Sel_z is $tan(\angle(a, s, b)/4)$. Note that the share price is in the range [0, 1] and it is an increasing function with respect to $\angle(a, s, b)$.
- Share price changes as the demand for the share changes. If share price increases for the seller Sel_z then $P(Sel_z)$ are also changed in such a way that the angle $\angle(a, s, b)$ gets bigger and vice versa.
- Share price changes for every interaction between a seller and any buyer. The rate of change of share price is uniform, i.e., increase(or decrease) for all possible interactions among the sellers and the buyers.

Now we formulate the reputation of a buyer using another circle whose center is on the circumference of the market circle. The reputation of a buyer is the length of the radius of this circle. Next, we formulate the association between the buyer who has provided a positive review for a seller and the reputation of the seller. We do so by using two lines (lines L_a and L_b as shown in Fig. 1b). These two lines change the length of radius of the buyer's circle as the reputation of the corresponding seller changes.

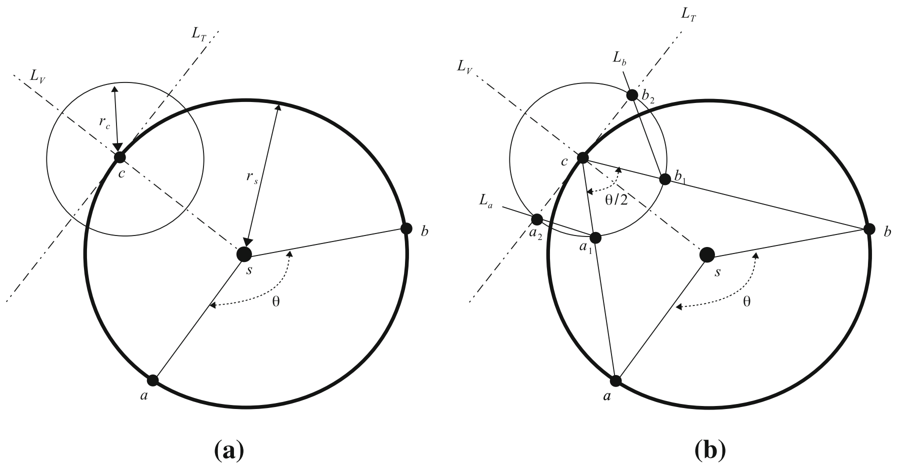


Fig. 1 **a** Market circle is the circle with center c and radius r_s. Seller points for a seller Sel_j is denoted as the points a and b. The share angle is θ. The buyer circle for Buy_i is illustrated with center c and radius r_c. L_T and L_V illustrate the buyer lines. **b** The seller lines are shown as the lines L_a and L_b as Buy_i buys a share of Sel_i

Definition 2 (*Share market and buyers*)

- Each buyer Buy_i is assigned (a) a point $P(Buy_i)$ on the circumference of $\mathbb{S}$ and (b) a circle $\mathbb{C}$ with center $P(Buy_i)$ and radius is r_c. The radius r_c is equal for all buyers who enter the share market. The reputation of the buyer Buy_i is the radius of the circle $\mathbb{C}$. The circle $\mathbb{C}$ is referred as the buyer circle.
- Additionally, each buyer is associated with two lines. L_V and L_T. L_V is the line from s(center of the market circle) through c (center of the buyer circle) and L_T is the line through c which is vertical to L_V. We refer these lines as the buyer lines.

Definition 3 (*Buying a share*) If a buyer Buy_i buys a share for the seller Sel_j with seller points a and b then we assign the following lines with Buy_i:

- The lines are L_a and L_b (refer to Fig. 1b).
- L_a is the line from a_1 to a_2 where a_1 is the first point of intersection between the circumference of the buyer circle $\mathbb{C}$ and the line from a to the center of the buyer circle c and a_2 is the first point of intersection between the circumference of $\mathbb{C}$ and the line L_T.
- Similarly we construct the line L_b.
- These two lines is referred as share lines of the buyer Buy_i for the seller Sel_j.

The change in the share price of a seller impacts the reputation of all buyers who had bought its shares. The procedure for such a change is as follows: Say, buyer Buy_i (with buyer circle $\mathbb{C}$ and buyer point c) had bought a share of seller Sel_j and the share price of Sel_j changes from $\tan\angle(a,s,b)/4$ to $\tan\angle(a',s,b')/4$.

- First we find the points a'_1 and b'_1. a'_1 (b'_1) is the first point of intersection between the line from a' and c (from b' and c) and $\mathbb{C}$.
- Next from a'_1 (b'_1) we draw a line L'_a (L'_b) parallel to share line L_a (L_b) of the buyer Buy_i for the seller Sel_j.
- Let a'_2 be the point of intersection of the line L'_a and the buyer line L_T.
- We redraw the buyer circle $\mathbb{C}$ with center c and radius as the length of the line segment from c to a'_2.

Note that, if the share price of Sel_j increases then the radius of the buyer circle also increases and vice versa. We illustrate the above procedure in Fig. 2. The next ingredient of the reputation management system is the condition that decides if a buyer is allowed to buy a share for a particular seller. Such a decision is taken by the reputation management system as follows: Say buyer Buy_i wants to buy the share of the seller Sel_j with share points a and b (refer to Fig. 3).

- Let d is a positive rational number less than r_c. d will be referred as the allowance factor. d remains the same for all buyers.
- Find the point z on the buyer line L_V at a distance d from c inside the market circle.

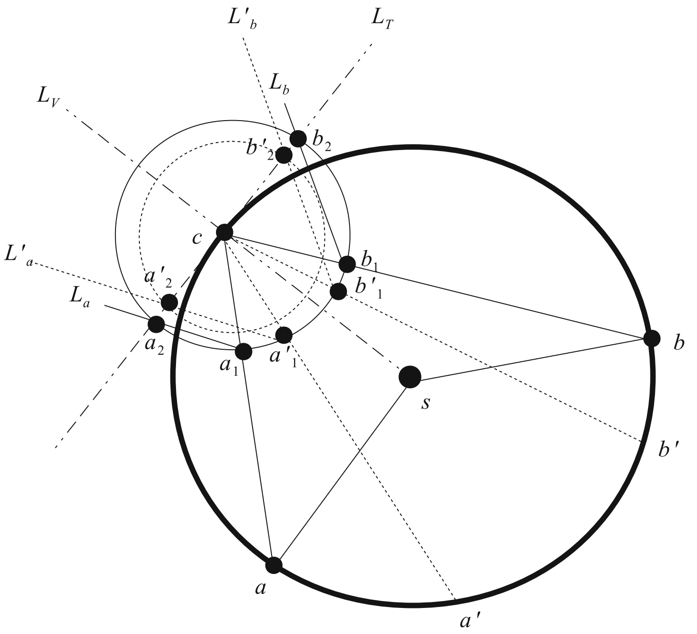
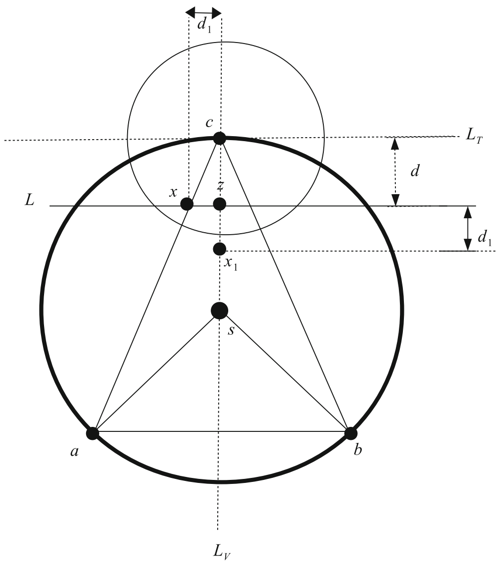


Fig. 2 Share price decreases from (a, b) to (a', b'). This impacts the reputation of the buyer who had bought its share. As shown the in figure, buyer circle also shrinks

Fig. 3 The procedure to decide if a buyer is allowed to buy a share from a particular seller and so to produce a feedback



- Draw a perpendicular line through z w.r.t L_V. Call it L. Say, this line intersects the lines ac at x. Let d_1 be the length of the line segments zx.
- Find the points x_1 on L_V at the distance $d + d_1$ from c.

- Buy_i can buy the share of Sel_j if x_1 resides inside its buyer circle. As illustrated in Fig. 4, x_1 resides outside the buyer circle. Hence the buyer is not allowed to buy the share. This means, a buyer with a low reputation can not affect the reputation of a seller with high reputation.

4 Analytical results

4.1 Interaction model

In this section, we discuss a model of interaction among the buyers and the sellers which is used in the analysis of the reputation management mechanism. There are two types of sellers. Let *Sel* be the set of all sellers.

- $B(Sel) \subset Sel$ and $G(Sel) \subset Sel$ are the sets of bad and good sellers respectively. The good sellers always sell good products and the bad sellers always sell bad products.
- $B(Sel) \cup G(Sel) = Sel$ and $B(Sel) \cap G(Sel) = \emptyset$

Let *Buy* be the set of all buyers. There are four types of buyers.

- $R(Buy)$ be the set of rational buyers. The rational buyers always report fair feedbacks.
- $IR(Buy)$ be the set of irrational buyers. The irrational buyers always provide unfair feedback.

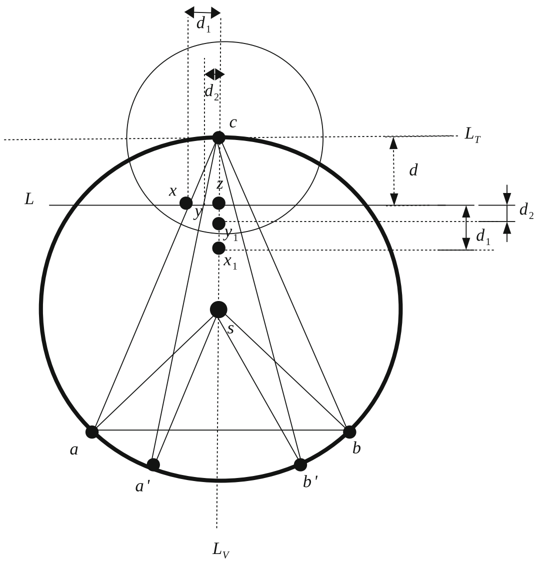


Fig. 4 Note that the buyer can buy the share for the seller with seller points (a', b') as y_1 resides inside its buyer circle but it can not do the same for the seller with seller points (a, b) as x_1 resides outside its buyer circle

- $CR(Buy)$ be the set of critical buyers. The critical buyers do not send any report. We consider such behaviour as a negative feedback.[1] Hence critical buyers always provide negative feedback irrespective of the quality of the products of the sellers.
- $RAN(Buy)$ be the set of random buyers. The random buyers randomly decide on reporting fair and unfair feedbacks.
- $R(Buy) \cup IR(Buy) \cup CR(Buy) \cup RAN(Buy) = Buy$ and a buyer may belong to only one category.

The interaction model is as follows (shown in Algorithm 1):

1. First, we initialize the sets of buyers and sellers with different types. We assign a positive rational number to Inc that represent unit changes of share price.
2. At every round, each seller is paired with a buyer chosen uniformly at random provided that they have not interacted before.
3. In each interaction between the seller Sel_j and the buyer Buy_i if

   - Buy_i is allowed to buy the share of Sel_j,
   - Buy_i has interacted atleast $BLimit$ times,
   - Sel_j has interacted atleast $SLimit$ times

   then, we follow the following steps:

4. If Sel_j has provided a good service, i.e., it belongs to the set $G(Sel)$ then,

   (a) If the buyer, say Buy_i is rational then it buys the share. We denote the shares it has bought so far as the set $\{Hold[i,j]\}$. $Hold[i,j] = 0$ indicates that buyer Buy_i has not bought a share from Sel_j. We change $Hold[i, j]$ to 1. $Sold[j]$ will indicate the number of shares sold by the seller Sel_j. We increase $Sold[j]$ by 1.
   (b) If the buyer is irrational then it refuses to buy the share. We denote the shares it has refused so far as the set $\{NoHold[i,j]\}$. $NoHold[i,j] = 0$ indicates that buyer Buy_i has not refused a share from Sel_j. We change $NoHold[i, j]$ to 1. We decrease $Sold[j]$ by 1.
   (c) If the buyer is critical then we change $NoHold[i, j]$ to 1 and decrease $Sold[j]$ by 1.
   (d) If the buyer is random then we flip a coin. If it is head then we set $Hold[i, j]$ to 1 and increase $Sold[j]$ by 1. Otherwise, we set $NoHold[i, j]$ to 1 and decrease $Sold[j]$ by 1.

5. If the seller had provided a bad service, i.e., $Sel_j \in B(Sel)$ then we follow the following procedure:

[1] Our RMM uses specific user interface for buyers where the buyer have to explicitly choose between ignoring the task to submit feedback or chooses not to buy shares, i.e., expresses a negative feedback.

   (a) If the buyer is rational then it refuses to buy the share. We change *NoHold*[*i*, *j*] to 1. We decrease *Sold*[*j*] by 1.
   (b) If the buyer is irrational then it buys the share. We change *Hold*[*i*, *j*] to 1. We increase *Sold*[*j*] by 1.
   (c) If the buyer is critical or random the procedure remains same as described in step 4.

6. If at step 3, only the conditions, Buy_i has interacted *BLimit* times and Sel_j has interacted *SLimit* times, do not satisfy then we follow the step 4 and 5 without modifying the parameter *Sold*. Instead, we increase a parameter *Scounter* to record the number of times a seller had interacted. Once $Scounter > SLimit$ we record the sale of shares sold by a seller using the parameter *Sold*.
7. At the end of each round we follow the following procedure:
   (a) For each buyer, if it has interacted more than *BLimit* times then we adjust its reputation using the changes of share prices for the shares it has bought so far (Procedure is shown in Algorithm 1).
   (b) For each seller, if it has interacted more than *SLimit* times then we adjust its share price with the changes in *Sold* parameter, if the change is Δ then its share price is changed as $Inc * \Delta$.

Note that, we change the reputation of the buyer and the seller only after they participate in a fixed number of interactions. A buyer with reputation r is allowed to review a seller with seller angle 4θ if $d(1 + \tan(\theta)) \leq r$. The parameters used in the following algorithms are as follows:

Parameter	Description
$G(Sell) \subset Sell$	Good sellers
$B(Sell) \subset Sell$	Good sellers
$R(Buy) \subset Buy$	Rational buyers,
$IR(Buy) \subset Buy$	Irrational buyers
$CR(Buy) \subset Buy$	Critical buyers
$RAN(Buy) \subset Buy$	Random buyers
$NoHold = \cup_{i,j}\{0, 1\}$	Refusal to buy shares
$Hold = \cup_{i,j}\{0, 1\}$	Acceptance to buy shares
$SLimit \in \mathbb{R}^+$	Sale limit
$BLimit \in \mathbb{R}^+$	Buyer limit
$Scounter \in \cup_{i \in [1:m]} Scounter_i$	Sale counter
$Sold = \cup_{i \in [1,M]} Sold_i$	Number of sold shares

Parameter	Description
$SRep = \cup_{i\in[1,m]} SRep_i$	Sellers reputation
$BRep \cup_{i\in[1,n]} BRep_i$	Buyers reputation
$d \in \mathbb{R}^+$	Allowance factor
$SDegree = \cup_{i\in[1:m]} SDegree_i$	Seller angle
$Old - SDegree$	*SDegree* of previous round
$Inc \in \mathbb{R}^+$	Unit change of Seller angle

Algorithm 1: Model of interaction among the buyers and the sellers.

```
Data: Set of n buyers Buy and m sellers Sel
Result: Interaction model
begin
    for Each round do
        OldSold ← Sold
        OldSDegree ← SDegree
        S ← Sample m buyers from Buy uniformly at random
        for Each Seller Sel_i do
            Buy_x ← Choose i'th buyer in S
            if Hold[x, i] == 0 then
                if Sel_i ∈ G(Sel) then
                    Update Sold,Hold,NoHold from the procedure described in
                    Algorithm-2.
                if Sel_i ∈ B(Sel) then
                    Update Sold,Hold,NoHold from the procedure described in
                    Algorithm-3.
        SRep ← SChange(OldSold, Sold)
        BRep ← BChange(OldSDegree, Hold, NoHold)
```

Algorithm 2: Part-1 of Algorithm-1

Data: Part-1 of Algorithm 1
Result:
begin
    Interaction between the buyer Buy_x and a good seller Sel_i
    if $Buy_x \in R(Buy)$ $And\ Allow(Buy_x, Sel_i) == Yes$ **then**
        $Hold[x, i] = 1$
        if $Scounter_i < SLimit$ **then**
            $Scounter_i + +$
        if $Scounter_i \geq SLimit$ **then**
            $Sold[x] + +$
    if $Buy_x \in IR(Buy)$ $And\ Allow(Buy_x, Sel_i) == Yes$ **then**
        $NoHold[x, i] = 1$
        if $Scounter_i < SLimit$ **then**
            $Scounter_i + +$
        if $Scounter_i \geq SLimit$ **then**
            $Sold[x] - -$
    if $Buy_x \in CR(Buy)$ $And\ Allow(Buy_x, Sel_i) == Yes$ **then**
        $NoHold[x, i] = 1$
        if $Scounter_i < SLimit$ **then**
            $Scounter_i + +$
        if $Scounter_i \geq SLimit$ **then**
            $Sold[x] - -$
    if $Buy_x \in RAN(Buy)$ $And\ Allow(Buy_x, Sel_i) == Yes$ **then**
        Buy_x decides randomly whether to buy or not **if** Buy_x *decides to buy* **then**
            $Hold[x, i] = 1$
            if $Scounter_i < SLimit$ **then**
                $Scounter_i + +$
            if $Scounter_i \geq SLimit$ **then**
                $Sold[x] + +$
        else
            $NoHold[x, i] = 1$
            if $Scounter_i < SLimit$ **then**
                $Scounter_i + +$
            if $Scounter_i \geq SLimit$ **then**
                $Sold[x] - -$

Algorithm 3: Part-2 of Algorithm-1

Data: Part-2 of Algorithm 1
Result:
begin
  if $Buy_x \in R(Buy)$ *And* $Allow(Buy_x, Sel_i) == Yes$ **then**
    $NoHold[x, i] = 1$
    if $Scounter_i < SLimit$ **then**
      $Scounter_i + +$
    if $Scounter_i \geq SLimit$ **then**
      $Sold[x] - -$
  if $Buy_x \in IR(Buy)$ *And* $Allow(Buy_x, Sel_i) == Yes$ **then**
    $Hold[x, i] = 1$
    if $Scounter_i < SLimit$ **then**
      $Scounter_i + +$
    if $Scounter_i \geq SLimit$ **then**
      $Sold[x] + +$
  if $Buy_x \in CR(Buy)$ *And* $Allow(Buy_x, Sel_i) == Yes$ **then**
    Same as the described in Algorithm-2
    $NoHold[x, i] = 1$
    if $Scounter_i < SLimit$ **then**
      $Scounter_i + +$
    if $Scounter_i \geq SLimit$ **then**
      $Sold[x] - -$
  if $Buy_x \in RAN(Buy)$ *And* $Allow(Buy_x, Sel_i) == Yes$ **then**
    Same as the described in Algorithm-2
    Buy_x decides randomly whether to buy or not **if** Buy_x *decides to buy* **then**
      $Hold[x, i] = 1$
      if $Scounter_i < SLimit$ **then**
        $Scounter_i + +$
      if $Scounter_i \geq SLimit$ **then**
        $Sold[x] + +$
    else
      $NoHold[x, i] = 1$
      if $Scounter_i < SLimit$ **then**
        $Scounter_i + +$
      if $Scounter_i \geq SLimit$ **then**
        $Sold[x] - -$

Algorithm 4: BChange

```
Data: Hold, NoHold, SDegree and Old − SDegree
Result: Modification of buyer's reputation.
begin
    for Each buyer Buy_i do
        if ∑Hold[i,] + ∑NoHold[i,] > BLimit then
            change1 ← 0
            change2 ← 0
            for Each seller Sel_j such that Hold[i,j] = 1 do
                if Old − SDegree_j < SDegree_j then
                    Increase Change1 using Equation 1
                if Old − SDegree_j > SDegree_j then
                    Decrease Change1 using Equation 1
            for Each seller Sel_j such that NoHold[i,j] = 1 do
                if Old − SDegree_j < SDegree_j then
                    Decrease Change2 using Equation 1
                if Old − SDegree_j > SDegree_j then
                    Increase Change2 using Equation 1
            Brep_i = Brep_i + Change1/∑Hold[i,] + Change2/∑NoHold[i,]
```

Algorithm 5: SChange

```
Data: Scounter, Sold, Old − Sold, Inc
Result: Modification of the seller's reputation.
begin
    for Each seller Sell_i do
        if Scounter > SLimit then
            SDegree_i = SDegree_i + Inc * (Sold[i] − Old − Sold[i])
```

Algorithm 6: Allow

```
Data: Buy_x, Sel_i
Result:
begin
    d ← Allowance factor
    4θ ← Seller angle of Sel_i
    Brep_x reputation of Buy_x
    p ← d(1 + tan(θ))
    if p ≤ Brep_x then
        Return('Yes')
    else
        Return('No')
```

4.2 Analysis

First, we calculate the change in a buyers reputation if the reputation of the seller (and the buyer had interacted with this seller) changes.

Lemma 1 *If a buyer Buy_i has bought the share of the seller Sel_i and the share angle of Sel_i changes from $4\theta_1$ to $4\theta_2$ then the change in the radius of Buy_i's buyer's circle is*

$$\frac{r * \cos\theta_2 * (1 - \sin\theta_1)}{\cos\theta_1} - r + r * \sin(\theta_2) \tag{1}$$

where r is the radius of the buyer circle for Buy_i.

Proof The scenario is illustrated in Fig. 5. The share angle changes from $4\theta_1$ to $4\theta_2$ as the share points are changed from (A, B) to ($A1$, $B1$). Note that the angles $\angle A1, C, S$ and $\angle A, C, S$ are θ_2 and θ_1 respectively. The share lines are changed from (z, a) to ($z1$, $a1$). Hence the change in the radius of the buyer's circle is the length of the line segment $zz1$. We calculate the length of the line segment $zz1$ as follows: Note that, $\angle c, z_1, a_1 = \angle c, z, a = \theta$. In the triangles Δabc and Δacz,

$$\frac{ab}{ac} = \sin\theta_1, \quad xc = ab = r * \sin\theta_1, \quad xz = r - r * \sin\theta_1 \tag{2}$$

Also,

$$\frac{cb}{ac} = \cos\theta_1, xa = cb = r\cos\theta_1 \tag{3}$$

In the triangle, $\Delta(xaz)$

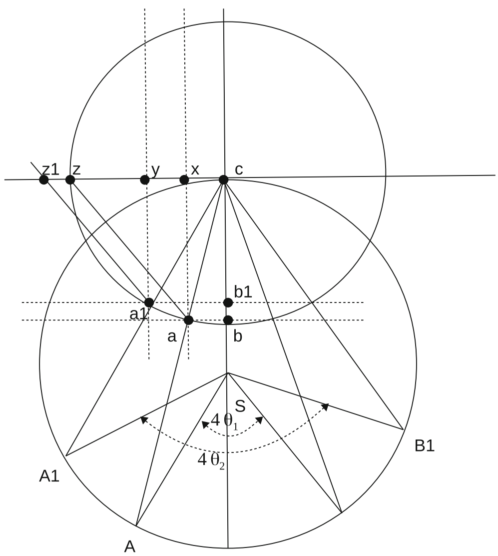


Fig. 5 Proof of Lemma 1

$$\tan\theta = \frac{xa}{xz} = \frac{r\cos\theta_1}{r - r*\sin\theta_1} = \frac{\cos\theta_1}{1-\sin\theta_1} \tag{4}$$

In the triangle, $\Delta(a1, b1, c)$

$$\cos\theta_2 = \frac{ya_1}{r}, ya_1 = r*\cos\theta_2 \tag{5}$$

$$\tan\theta = \frac{r*\cos\theta_2}{yz_1} = \frac{\cos\theta_1}{1-\sin\theta_1} \tag{6}$$

$$yz_1 = \frac{r*\cos\theta_2*(1-\sin\theta_1)}{\cos\theta_1} \tag{7}$$

In triangle $\Delta(a, y, c)$,

$$\tan(90-\theta_2) = \frac{r*\cos\theta_2}{yc} yc = \frac{r*\cos\theta_2}{\tan(90-\theta_2)} \tag{8}$$

Hence,

$$zy = r - \frac{r*\cos\theta_2}{\tan(90-\theta_2)}. \tag{9}$$

$$zz_1 = yz_1 - yz \tag{10}$$

$$\begin{aligned} zz_1 &= \frac{r*\cos\theta_2*(1-\sin\theta_1)}{\cos\theta_1} - r + \frac{r*\cos\theta_2}{\tan(90-\theta_2)} \\ zz_1 &= \frac{r*\cos\theta_2*(1-\sin\theta_1)}{\cos\theta_1} - r + \frac{r*\cos\theta_2*\sin(\theta_2)}{\cos(\theta_2)} \\ zz_1 &= \frac{r*\cos\theta_2*(1-\sin\theta_1)}{\cos\theta_1} - r + r*\sin(\theta_2) \end{aligned} \tag{11}$$

Note that, according to Eq. 11, a buyer's reputation decreases if the corresponding seller's reputation decreases (assuming it has provided a positive review for it) and vice versa. Next, we derive the condition which decides whether or not a buyer is allowed report its review about a seller. □

Lemma 2 *A buyer Buy_i is allowed to buy the share of a seller Sel_j if current radius of Buy_i's buyer's circle is at least $d + d*\tan\theta$ where d is the allowance factor and seller angle of Sel_j is 4θ.*

Proof Please refer to Fig. 3 for the explanation.

$$\tan\theta = \frac{d_1}{d}, d_1 = d*\tan\theta \tag{12}$$

Hence, A buyer Buy_i is allowed to buy the share of a seller Sel_j if current radius of Buy_i's buyer's circle is at least $d + d*\tan\theta$ where d is the allowance factor and seller angle of Sel_j is 4θ. □

Lemma 3 *Let d be the allowance factor, r be the initial reputation of all buyers and $4\theta_1$ be the initial seller angle for all sellers. It is required that $r > d(1 + \tan(\theta_1))$ so that the buyers can provide review for the sellers.*

In the next Lemma we derive the conditions whose satisfaction guarantee that the irrational, critical and random buyers can not manipulate the reputation management system. In this lemma, we assume that the number of interaction among the sellers and the buyers is such that no buyer is yet disallowed to report a feedback based on the allowance factor. In fact, we intend to find such a number of steps when the allowance factor does not allow the irrational, critical and random buyers to report their review.

Lemma 4 *Let an e-marketplace has the following demography:*

- *There are P good sellers and Q bad sellers where $\alpha = P/Q$.*
- *There are $n/4 + \epsilon$ rational buyers, n / 4 irrational buyers, n / 4 critical buyers and n / 4 random buyers where $\epsilon > 0$.*

In this e-marketplace,

- *an irrational buyer will be eventually not allowed to provide a review for a good seller if:*

$$d(1 + \tan(\theta_2)) < r - \frac{r}{1+\alpha}(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2'))$$

- *A rational buyer will be eventually allowed to provide a review for a good seller if*

$$d(1 + \tan(\theta_2)) < r + \frac{r}{1+\alpha}[\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2')]$$

- *A rational buyer will be eventually allowed to provide a review for a bad seller if*

$$d(1 + \tan(\theta_2')) < r + \frac{r}{1+\alpha}[\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2')]$$

- *a critical buyer will be eventually not allowed to provide a review for a good seller if*

$$d(1 + \tan(\theta_2)) > r - \frac{r}{1+\alpha}(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) + (x\cos\theta_2' - 1 + \sin\theta_2'))$$

- *a random buyer will be eventually not allowed to provide a review for a good seller if*

$$d(1 + \tan(\theta_2)) > r$$

where $x = \frac{1-\sin\theta_1}{\cos\theta_1}$ $4\theta_1$ *is the initial seller angle for all sellers,* r *is the initial reputation of all buyers,* d *is the allowance factor and* $SLimit \geq z > BLimit$ *is a positive integer.*

Proof In any interaction, the probabilities of interacting with a rational, irrational, random and critical buyer are as follows:

$$P = \begin{cases} \frac{n/4+\epsilon}{n+\epsilon} & \text{Rational buyer} \\ \frac{n/4}{n+\epsilon} & \text{Irrational buyer} \\ \frac{n/4}{n+\epsilon} & \text{Random buyer} \\ \frac{n/4}{n+\epsilon} & \text{Critical buyer} \end{cases}$$

Thus in z interactions, the expected interaction of any seller is as follows:

$$E = \begin{cases} (z - SLimit)\frac{n/4+\epsilon}{n+\epsilon} & \text{Rational buyer} \\ (z - SLimit)\frac{n/4}{n+\epsilon} & \text{Irrational buyer} \\ (z - SLimit)\frac{n/4}{n+\epsilon} & \text{Random buyer} \\ (z - SLimit)\frac{n/4}{n+\epsilon} & \text{Critical buyer} \end{cases}$$

Hence the change in the seller angle of a good seller Sel_i is as follows:

$$\begin{aligned} &(z - SLimit)\left[\frac{n/4+\epsilon}{n+\epsilon} - \frac{n/4}{n+\epsilon} - \frac{n/4}{n+\epsilon} - \frac{1}{2}\frac{n/4}{n+\epsilon} + \frac{1}{2}\frac{n/4}{n+\epsilon}\right] \\ &= (z - SLimit)\frac{\epsilon - n/4}{n+\epsilon} \end{aligned} \tag{13}$$

And, the change in the seller angle of a bad seller Sel_j is as follows:

$$\begin{aligned} &(z - SLimit)\left[-\frac{n/4+\epsilon}{n+\epsilon} + \frac{n/4}{n+\epsilon} - \frac{n/4}{n+\epsilon} - \frac{1}{2}\frac{n/4}{n+\epsilon} + \frac{1}{2}\frac{n/4}{n+\epsilon}\right] \\ &= -(z - SLimit)\frac{n/4+\epsilon}{n+\epsilon} \end{aligned} \tag{14}$$

The reputations of the sellers after z steps are as follows:

$$\left[\theta_1 + (z - S\,Limit)\left[\frac{\epsilon - n/4}{n + \epsilon}\right]\right] \quad \text{Good seller} \tag{15}$$

$$\left[\theta_1 + (z - S\,Limit)\left[-\frac{n/4 + \epsilon}{n + \epsilon}\right]\right] \quad \text{Bad seller} \tag{16}$$

Based on the new seller angles of the good and the bad seller, let:

$$\theta_2 = \left[\theta_1 + (z - S\,Limit)\left[\frac{\int - n/4}{n + \int}\right]\right] \Big/ 4 \quad \text{Good seller}$$

$$\theta_2' = \left[\theta_1 + (z - S\,Limit)\left[-\frac{n/4 + \int}{n + \int}\right]\right] \Big/ 4 \quad \text{Bad seller}$$

Also, a rational buyer buys $\frac{\alpha}{1+\alpha}$ shares from the good sellers and refuses to buy $\frac{\alpha}{1+\alpha}$ shares from bad sellers. Similarly, an irrational buyer, buys $\frac{1}{1+\alpha}$ shares from bad sellers and refuses to buy $\frac{\alpha}{1+\alpha}$ shares from good sellers. Reputation of a rational buyer, say $Rep(Buy_i)$, after z interactions is as follows:

$$\begin{aligned} Rep(Buy_i) &= r + \frac{r\alpha}{1+\alpha}\left[\frac{\cos\theta_2(1-\sin\theta_1)}{\cos\theta_1} - 1 + \sin\theta_2\right] \\ &\quad - \frac{r}{1+\alpha}\left[\frac{\cos\theta_2'(1-\sin\theta_1)}{\cos\theta_1} - 1 + \sin\theta_2'\right] \\ &= r + \frac{r\alpha}{1+\alpha}[x\cos\theta_2 - 1 + \sin\theta_2] - \frac{r}{1+\alpha}[x\cos\theta_2' - 1 + \sin\theta_2'] \\ &= r + \frac{r}{1+\alpha}[\alpha(x\cos\theta_2 - 1 + \sin\theta_2) \\ &\quad - (x\cos\theta_2' - 1 + \sin\theta_2')] \end{aligned} \tag{17}$$

Reputation of an irrational buyer, say $Rep(Buy_j)$, after z interactions is as follows:

$$\begin{aligned} Rep(Buy_j) &= r - \frac{r\alpha}{1+\alpha}\left[\frac{\cos\theta_2(1-\sin\theta_1)}{\cos\theta_1} - 1 + \sin\theta_2\right] \\ &\quad + \frac{r}{1+\alpha}\left[\frac{\cos\theta_2'(1-\sin\theta_1)}{\cos\theta_1} - 1 + \sin\theta_2'\right] \\ &= r - \frac{r\alpha}{1+\alpha}[x\cos\theta_2 - 1 + \sin\theta_2] \\ &\quad + \frac{r}{1+\alpha}[x\cos\theta_2' - 1 + \sin\theta_2'] \\ &= r - \frac{r}{1+\alpha}(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) \\ &\quad - (x\cos\theta_2' - 1 + \sin\theta_2')) \end{aligned} \tag{18}$$

Similarly, the reputation of an critical buyer, say $Rep(Buy_k)$, after z interactions is as follows:

$$Rep(Buy_j) = r - \frac{r}{1+\alpha}(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) + (x\cos\theta_2' - 1 + \sin\theta_2')) \tag{19}$$

And the reputation of a random buyer after z interactions is r. Following from Lemma 2, the claims in this Lemma are true. □

Also, in the above scenario (when the number of interactions are too few so that the allowance factor does not prevent the irrational, critical and random buyers to report feedback) we observe the following:

1. The reputation of all good sellers remains the same. This is because a pair of buyer and seller is paired uniformly at random to interact.
2. The reputation of all bad sellers remains the same. This is because a pair of buyer and seller is paired uniformly at random to interact.
3. The reputation of all sellers decrease. Note that, Eqs. 15 and 16 are decreasing with respect to z.
4. But, the reputation of the good sellers are still better than the reputation of the bad sellers.

Lemma 5 *Reputation of rational buyers remains more than the reputation of irrational buyers if the following conditions are satisfied*:

- *If* $\alpha \geq 1$ *then, any small* θ_1 *should be enough,*
- *If* $\alpha < 1$ *then it must hold that* $\alpha > \frac{(x\cos\theta_2' - 1 + \sin\theta_2'))}{(x\cos\theta_2 - 1 + \sin\theta_2)}$,

where α *is the ratio between the number of good and bad sellers,* θ_1 *is the initial seller angle,* θ_2 *and* θ_2' *are the quarter of good and bad seller angles after* z *steps and* $x = (1 - sin(\theta_1))/cos(cos(\theta_1))$.

Proof After z steps, the reputation of rational and irrational buyers are shown in Eqs. 17 and 18. We need to show the following:

$$\begin{aligned}
& r + \frac{r}{1+\alpha}[\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2')] \\
& \quad > r - \frac{r}{1+\alpha}(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2')) \\
& \quad [\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2')] \\
& \quad > -(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2')) \\
& \quad \alpha(x\cos\theta_2 - 1 + \sin\theta_2) > (x\cos\theta_2' - 1 + \sin\theta_2'))
\end{aligned}$$

If we assume $\alpha > 1$ we need to show

$$(x\cos\theta_2 - 1 + \sin\theta_2) > (x\cos\theta_2' - 1 + \sin\theta_2'))$$

Note that $x = \frac{1-\sin(\theta_1)}{\cos(\theta_1)} \simeq 1$ for small θ_1. Hence we need to show,

$$(\cos\theta_2 + \sin\theta_2) > (\cos\theta_2' + \sin\theta_2')) \tag{20}$$

Note that $\theta_2 \in [0, 45]$. In this range, $(\cos y + \sin y)$ is an increasing function w.r.t y. Hence Eq. 20 holds as $\theta_2 > \theta_2'$.

Now, assume that $\alpha < 1$. We need to show:

$$\alpha > \frac{(x\cos\theta_2' - 1 + \sin\theta_2'))}{(x\cos\theta_2 - 1 + \sin\theta_2)}$$

□

4.3 Market makers

In this section, we analyse the choice of various parameters that ensures the conditions described in Lemma 4 are satisfied, i.e., we analyse the decision to be taken by the market makers. The parameters for the market maker are as follows:

- *Initial seller reputation* The initial reputation of the sellers depends on θ_1. Note that, we need to find appropriate value θ_1 to satisfy the conditions of Lemma 5. According to these conditions if we assume that the number of good sellers is at least same as the number of bad sellers then we need to set the value of θ_1 as small as possible, i.e., any small positive rational number. If we assume that the number of good sellers is less than the number of bad sellers then we may need to choose θ_1 in such a way that it satisfies second condition of Lemma 5.
- *Demography of the sellers* This the denoted as α. In this section we will analyse two decision making processes based on α. First we will assume that $\alpha < 1$ and second we will assume that $\alpha \geq 1$. It will be assumed that the market maker knows this parameter or such knowledge may reflect its confidence on the quality of sellers in the market. Based on such confidence on the quality of sellers, i.e., α, the market maker has to choose the parameter θ_1 as described before.
- *Allowance factor and initial buyer's reputation* These parameters are d and r respectively. According to Lemma 3, it is required that $d(1 + \tan(\theta_1)) < r$.

Now, we analyse the choice of parameters to satisfy Lemma 4. Our objective is shown in Fig. 6. We must choose the parameters in such a way that the following are achieved:

1. Reputation of the irrational buyers must be eventually lower than $d(1 + \tan(\theta_2))$ where $4\theta_2$ is the seller angle for good sellers. This will ensure that the allowance factor discards reviews from the irrational buyers.
2. The reputation of the rational buyers must be more than the irrational buyers so that they can provide review for good or bad sellers.

To satisfy the above conditions we show the following:

- In Lemma 6 we show that, the reputation of the buyers and the value of $d(1 + \tan(\theta))|\theta \in \{\theta_2, \theta_2'\}$ decreases.

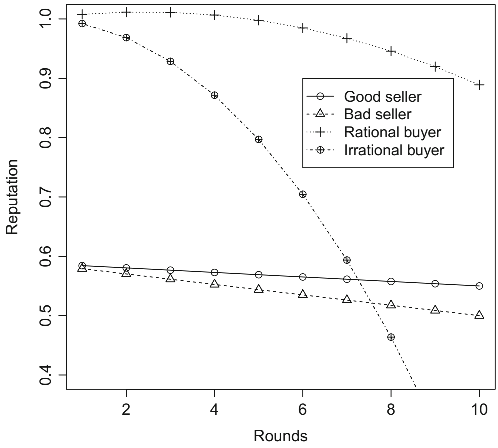


Fig. 6 Objective of the market maker to ensure that the allowance factor filters unfair reviews. We need to establish that, **a** the reputation of the irrational less than the same of the rational buyers and **b** the reputation of the irrational buyers is less than $d(1 + \tan(\theta_2))$ where $4\tan(\theta_2)$ is the seller angle of the good sellers

- We assign the initial reputation of the buyers is more than $d(1 + \tan(\theta_1))$.
- In Lemma 5 we show that, the reputation of the irrational buyers remains lower than the same for the rational buyers.
- In Lemma 6 we show that, the rate of decrease of reputation of the irrational buyers higher than the rate of change of $d(1 + \tan(\theta_2))$ where $4\theta_2$ is the seller angle for the good sellers.

Lemma 6 *The rate of decrease of reputation of the irrational buyers higher than the rate of change of* $d(1 + \tan(\theta_2))$ *where* $4\theta_2$ *is the seller angle for the good sellers if* $(1 + \alpha) > \frac{r}{d}$.

Proof The rate of change of $d(1 + \tan(\theta_2))$ w.r.t z is as follows:

$$\frac{d}{dz}(d + d\tan(\theta_2)) = .25d(1 + \tan^2(\theta_2))\left[\frac{\epsilon - n/4}{n + \epsilon}\right]$$

The rate of change of irrational buyer's reputation is as follows:

$$\frac{d}{dz}\left(r - \frac{r}{1+\alpha}(\alpha(x\cos\theta_2 - 1 + \sin\theta_2) - (x\cos\theta_2' - 1 + \sin\theta_2'))\right)$$
$$= \frac{-r}{1+\alpha}\left(\alpha\frac{d}{dz}(\theta_2)(-x\sin\theta_2 + \cos\theta_2) - \frac{d}{dz}(\theta_2')(-x\sin\theta_2' + \cos\theta_2')\right)$$

Also,

$$\frac{d}{dz}(\theta_2) = .25\left[\frac{\epsilon - n/4}{n + \epsilon}\right]\Bigg]. \tag{21}$$

$$\frac{d}{dz}(\theta_2') = .25\left[\left[-\frac{n/4 + \epsilon}{n + \epsilon}\right]\right]. \tag{22}$$

Hence we need to show:

$$
\begin{aligned}
&.25d(1+\tan^2(\theta_2))\left[\frac{\epsilon - n/4}{n+\epsilon}\right] \\
&\quad > \frac{-r}{1+\alpha}\left(\alpha.25\left[\frac{\epsilon - n/4}{n+\epsilon}\right]\right](-x\sin\theta_2+\cos\theta_2) \\
&\qquad - .25\left[\left[-\frac{n/4+\epsilon}{n+\epsilon}\right]\right](-x\sin\theta_2'+\cos\theta_2')) \\
&\quad d(1+\tan^2(\theta_2))[\epsilon - n/4] \\
&\quad > \frac{-r}{1+\alpha}(\alpha[\epsilon - n/4]](-x\sin\theta_2+\cos\theta_2) \\
&\qquad - [[-(n/4+\epsilon)]](-x\sin\theta_2'+\cos\theta_2'))
\end{aligned}
$$

For small ϵ

$$
\begin{aligned}
d(1+\tan^2(\theta_2)) &> \frac{-r}{1+\alpha}(\alpha(-x\sin\theta_2+\cos\theta_2) \\
&\quad - (-x\sin\theta_2'+\cos\theta_2')) \\
&\frac{d(1+\alpha)(1+\tan^2(\theta_2))}{r(-\alpha(-x\sin\theta_2+\cos\theta_2)+(-x\sin\theta_2'+\cos\theta_2'))} > 1 \\
&\frac{d(1+\alpha)(1+\tan^2(\theta_2))}{r(-\alpha(-x\sin\theta_2+\cos\theta_2)+(-x\sin\theta_2'+\cos\theta_2'))} > 1 \\
&\frac{(1+\alpha)(1+\tan^2(\theta_2))}{(-\alpha(-x\sin\theta_2+\cos\theta_2)+(-x\sin\theta_2'+\cos\theta_2'))} > \frac{r}{d}
\end{aligned}
$$

Now we calculate the minimum of the left-hand side as follows:

$$
\begin{aligned}
&Min\left(\frac{(1+\alpha)(1+\tan^2(\theta_2))}{(-\alpha(-x\sin\theta_2+\cos\theta_2)+(-x\sin\theta_2'+\cos\theta_2'))}\right) \\
&= 1+\alpha
\end{aligned}
$$

Note that, in the range $\theta \in [0, 45]$, minimum of $\tan(\theta)$ is 0, minimum and maximum of $cos(\theta) - sin(\theta)$ are 0 and 1 respectively. Also, if we need to keep θ_1 low. Hence $x = (1 - sin(\theta_1))/cos(\theta_1)$ becomes 1. □

Note that, so far we have established the conditions for which the allowance factor filters the irrational buyers. In the next Lemma we show that, after the allowance factor prevents the irrational buyers to review good sellers, the reputation of the rational buyers becomes more than the for the critical and random buyers.

Lemma 7 *Once the allowance factor prevents the irrational buyers to report feedback about a good seller, the reputation of rational buyers becomes more than the same of the critical and the random buyers.*

Proof Note that, the seller angle of good sellers becomes:

$$(z)\left[\frac{n/4+\epsilon}{n+\epsilon}-\frac{n/4}{n+\epsilon}-\frac{1}{2}\frac{n/4}{n+\epsilon}+\frac{1}{2}\frac{n/4}{n+\epsilon}\right]=z\frac{\epsilon}{n+\epsilon} \tag{23}$$

Let $\theta_3=(\theta_1+z\frac{\epsilon}{n+\epsilon})/4$. Now the slope of the equation $d+d\tan(\theta_3)$ is as follows:

$$\frac{d}{dz}(d+d\tan(\theta_3))=.25d(1+\tan^2(\theta_3))\left(\frac{\epsilon}{n+\epsilon}\right) \tag{24}$$

We need to show:

$$\begin{aligned}
&\frac{d}{dz}\left(r+\frac{r}{1+\alpha}(\alpha(x\cos\theta_3-1+\sin\theta_3)-(x\cos\theta_2'-1+\sin\theta_2'))\right)\\
&\quad>\frac{d}{dz}\left(r-\frac{r}{1+\alpha}(\alpha(x\cos\theta_3-1+\sin\theta_3)+(x\cos\theta_2'-1+\sin\theta_2'))\right)\\
&\quad\frac{d}{dz}\left(\frac{r}{1+\alpha}(\alpha(x\cos\theta_3+\sin\theta_3)\right)>0\\
&\quad\frac{d(\theta_3)}{dz}\left(\frac{r}{1+\alpha}(\alpha(-x\sin\theta_3+cos\theta_3)\right)>0
\end{aligned}$$

Note that $(-x\sin\theta_3+cos\theta_3)>0$ and $\frac{d}{dz}(d+d\tan(\theta_3))>0$. Hence the above holds. Reputation of random buyers remain the same. But for rational buyers it increases. □

We summarize the market makers decisions as follows:

Parameter	Value
r/d	>1
r/d	$<1+\alpha$
θ_1	Small positive rational number if $\alpha\geq 1$
θ_1	should satisfy $\alpha>\frac{(x\cos\theta_2'-1+\sin\theta_2'))}{(x\cos\theta_2-1+\sin\theta_2)}$ if $\alpha<1$

Theorem 1 *The reputation management model encourages the rational behaviour of the buyers.*

Proof We need to show that, the reputation management model ensures that the rational buyers get better reputation than other type of buyers. In Lemmas 5 and 6 we have established the conditions whose satisfaction guarantees that (a) the allowance factor will prevent the irrational buyers from reporting feedback about the good sellers (b) once irrational buyers are not allowed to review good sellers, the reputation of the rational buyers becomes more than the same for all other types of buyers. □

Theorem 2 *Good sellers get better reputation than bad sellers.*

Proof Following from Eqs. 24 and 22, note that rate of increment of the share price of the good sellers is more than the same for the bad sellers. □

4.4 Collusion

We consider the following collusion scenario: A bad seller colludes with a group of buyers to provide good review so that its reputation remains same as good sellers. We calculate the number of buyers with whom the seller needs to collude. Such number of buyers, before the allowance factor filters review from irrational or critical or random buyers is as follows: Using Eqs. 17 and 18 we get:

$$(z - SLimit)\left[\frac{\epsilon - n/4}{n+\epsilon}\right] - \left[-\frac{n/4+\epsilon}{n+\epsilon}\right]\Bigg] = (z - SLimit)\left[\frac{2\epsilon}{n+\epsilon}\right]$$

Such a number of buyers, after the allowance factor filters review from irrational or critical or random buyers is as follows: Using Eqs. (24, 17 and 18) we get:

$$(z)\left[\frac{\epsilon}{n+\epsilon}\right] - \left[-\frac{n/4+\epsilon}{n+\epsilon}\right]\Bigg] = (z)\left[\frac{2\epsilon + n/4}{n+\epsilon}\right]$$

Note that, if we change the seller angle per review slowly, the number of positive reviews (using collusion) will be very high. We assume that the buyers are allowed to provide review after any actual transaction between the buyer and the seller. For example, a hotel booking website allows to review the hotel accommodation few days after the after the services has been provided. If the seller bears the cost of such transaction, (i.e., it pays the buyers to book hotels) then it should be financially infeasible to collude.

5 Experimental results

We use Algorithm 1 to simulate the e-marketplace. There are 40 sellers. And around 400 buyers. We use the following parameters.

Parameter	Value	Parameter	Value
G(*Sell*)	[15, 30]	*B*(*Sell*)	[10, 25]
R(*Buy*)	110	*IR*(*Buy*)	100
CR(*Buy*)	100	*RAN*(*Buy*)	100
SLimit	2	*BLimit*	2
d	3.5	*r*	4
θ_1	2.5	*Inc*	1

Figures 7 and 8 show the simulation result where the number of good and bad sellers are equal and the number of rational buyers are 110 and all other types of

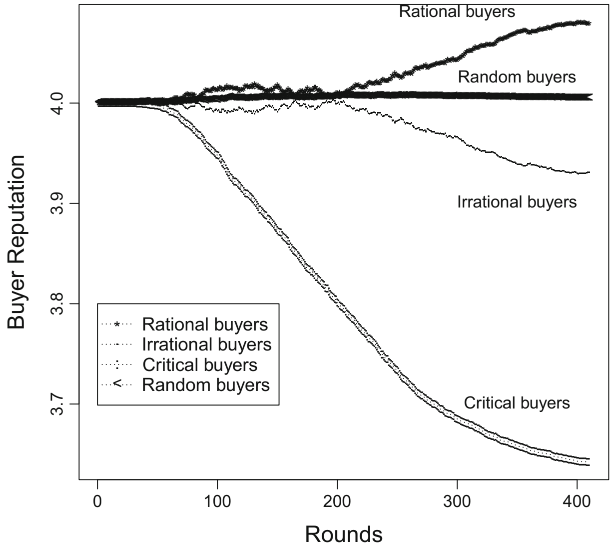


Fig. 7 20 good and 20 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

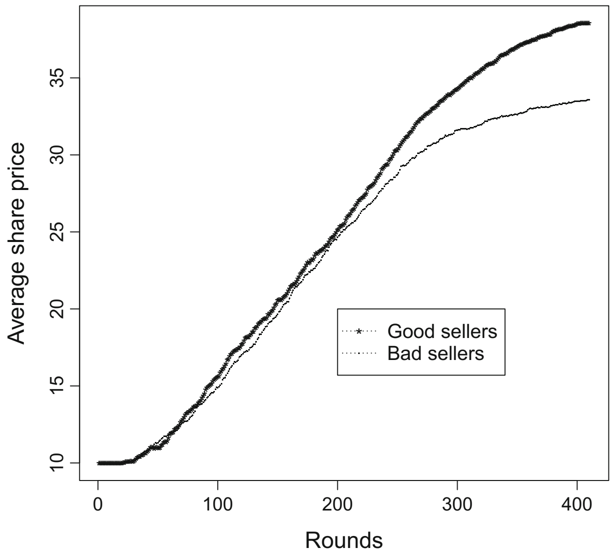


Fig. 8 20 good and 20 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

buyers are 100. It clearly shows that the reputation of the rational buyers increases and the reputation of the irrational buyer decreases. It also shows that the share price, i.e., the reputation of the good sellers increased more than the reputation of the bad seller.

Next, we study the effect of increasing the number of good sellers in the e-marketplace. In Figs. 9 and 10 we show the result with 25 good and 15 bad sellers (and 110 rational buyers, 100 irrational buyers,100 critical buyers, 100 random buyers). In Figs. 11 and 12 we show the result with 30 good and 10 bad sellers (and 110 rational buyers, 100 irrational buyers,100 critical buyers, 100 random buyers). It clearly shows that with the increment of the number of good sellers, the reputation of the rational buyers increases and the reputation of the good sellers remains more than the reputation of the bad sellers.

Next, we study the effect of decreasing the number of good sellers in the e-marketplace. In Figs. 13 and 14 we show the result with 15 good and 25 bad sellers (and 110 rational buyers, 100 irrational buyers,100 critical buyers, 100 random buyers). It shows that the reputation of the rational buyers are better than others and reputation of the good sellers are better than bad sellers. But it takes more rounds before the reputation of the rational buyers gets better w.r.t the decrement of the number of good sellers. By decreasing the number of good sellers beyond 10, we found that the reputation of the rational buyers decreases and the same decreases for the good sellers.

Next, we perform experimental analysis with a real review dataset. We use the Amazon review dataset in the category 'Grocery and Gourmet Food' [15]. We only use 100000 reviews. In this data, there are 10791 sellers and 87063 buyers. The buyers give a rating between 0 to 5. A rating of 5 indicates the best review (positive review) and a rating of 0 indicates the worst review (negative review). Note that the experimental analysis of the reputation management mechanism requires knowledge of the behaviours of the seller and the buyers, i.e., whether a seller is good or bad and whether a buyer is rational or irrational or critical or random. From the

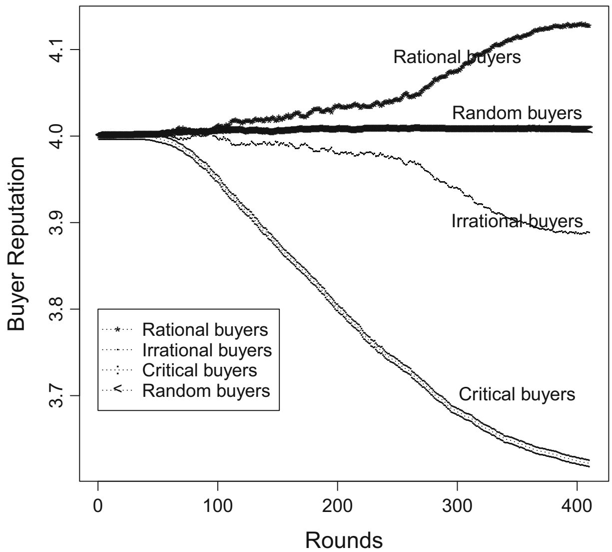


Fig. 9 25 good and 15 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

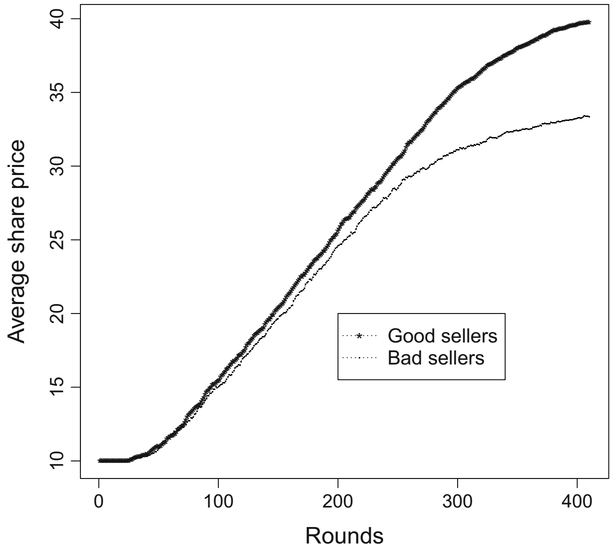


Fig. 10 25 good and 15 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

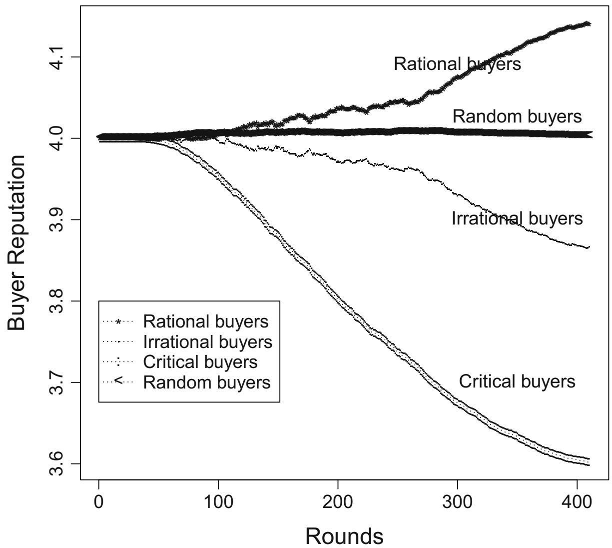


Fig. 11 30 good and 10 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

above mentioned dataset we identify the behaviours of the sellers as follows: First we calculate the average rating that each seller has received. Next, if the reputation of a seller is more than 4 then, we recognize the seller as a good seller otherwise we treat it as a bad seller. We found 8208 good sellers and 2583 bad sellers.

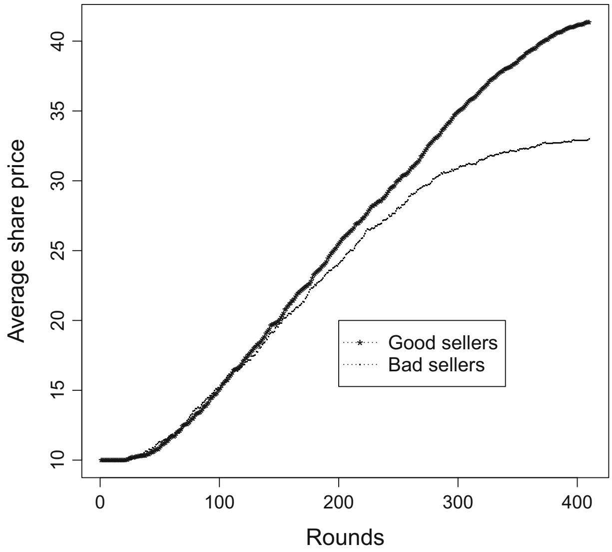


Fig. 12 30 good and 10 bad sellers. 110 rational buyers, 100 irrational buyers,100 critical buyers, 100 random buyers

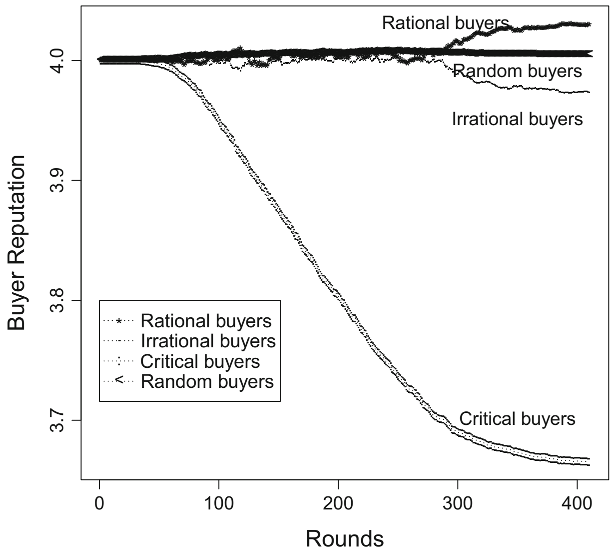


Fig. 13 15 good and 25 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

Next, we determine the behaviours of the buyers as follows[2] (Fig. 15 shows the selection of behaviours). For each buyer, (a) we calculate the total ratings of all

[2] We do not consider random buyers in this data as it is not clear how to identify them for this data.

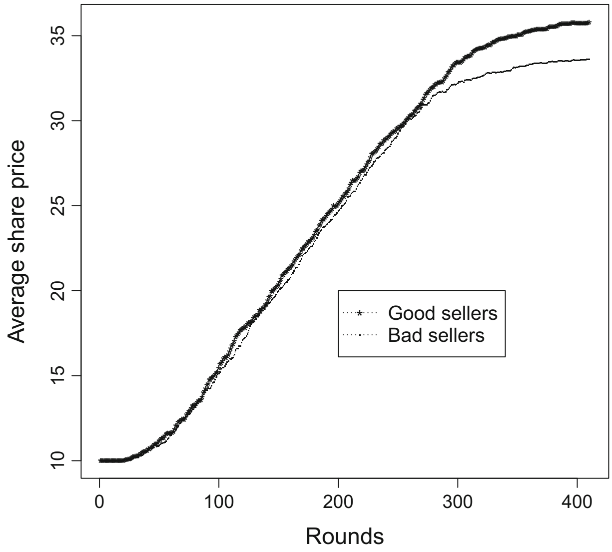


Fig. 14 15 good and 25 bad sellers. 110 rational buyers, 100 irrational buyers, 100 critical buyers, 100 random buyers

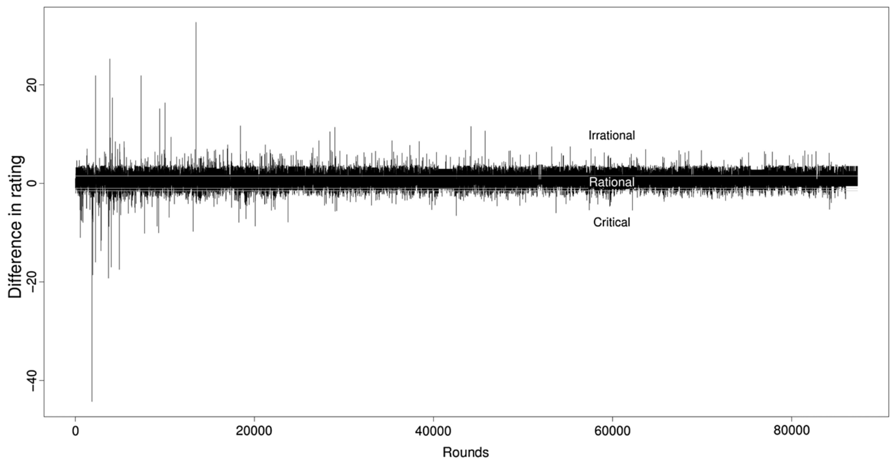


Fig. 15 Amazon dataset: There are 63,803 rational reviewers, 20,047 irrational reviewers and 3213 critical reviewers

sellers it had reviewed, denote it as d_1 and, (b) we calculate the total ratings it has given, denote it as d_2. Let $d_3 = d_1 - d_2$. A positive d_3 indicates that the buyer has provided good reviews (overall) where other buyers may have rated the same set of sellers with less ratings. A negative d_3 indicates that the buyer has provided bad reviews (overall) where other buyers may have rated the same set of sellers with more ratings. We identify the set of rational buyers if for each of them the value of d_3 is in the range $(-1, 1.5)$. We identify the set of irrational buyers if for each of

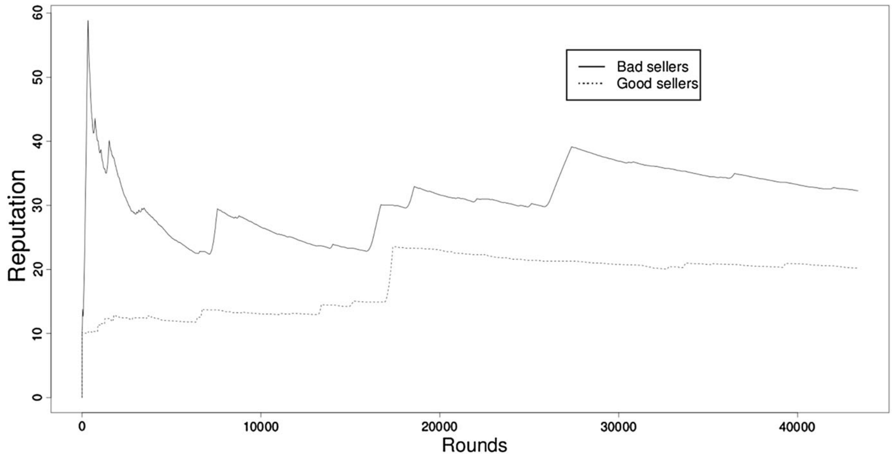


Fig. 16 Outcome of our experiment. It shows that the average reputation of the good sellers is higher than the same for the bad sellers

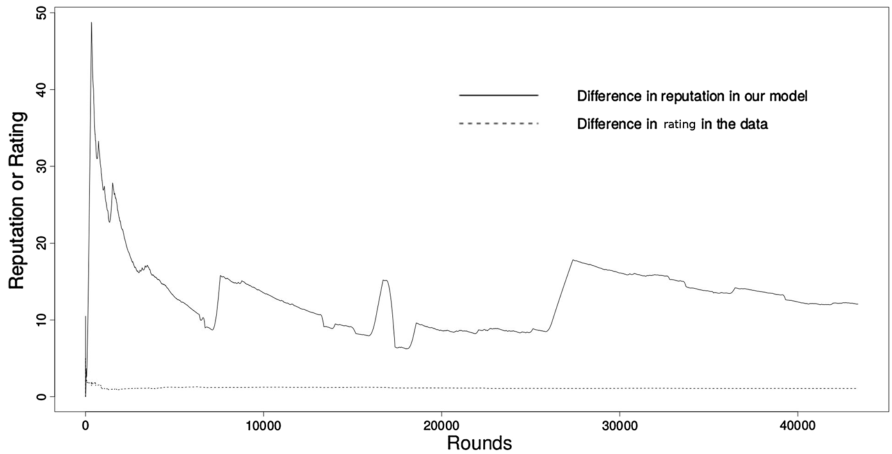


Fig. 17 A comparison between the outcome (reputation of the sellers) of our experiment and the ratings given in the dataset. It shows that using reputation it is easy to distinguish a good seller from a bad seller

them the value of d_3 is in the range $[-1, -1.5)$ or $[1.5, \infty]$. We identify the set of critical buyers if for each of them the value of d_3 is less or equal to $[-1.5)$. Following these steps, we get 63,803 rational reviewers, 20,047 irrational reviewers and 3213 critical reviewers.

We execute the simulation with the identified behaviour of the sellers and the buyers. Interactions between the buyers and the sellers are according to the interactions given in the dataset, i.e., we choose a seller to interact with a buyer in the given sequence of interactions in the dataset. Figure 16 shows the outcome of the experiment. It clearly shows that the average reputation of the good sellers are higher than the same for the bad sellers. Figure 17 shows the difference between the reputation(from our experiment) and rating (in the dataset) for the good and bad sellers. It clearly shows that the difference between the reputation of good and bad sellers is much higher than the difference between the rating of the good and the bad sellers. Hence using our model of reputation management mechanism it is easy to distinguish between a good and a bad seller.

6 Conclusion

In this paper we have developed a reputation management mechanism that exploits an association between the reputation of the buyers and the sellers to encourage the buyers to report fair feedbacks. We show that the reputation of the rational buyer remains more than the same for the irrational, critical or random buyers. Also we show that, the good seller gets better reputation than the bad sellers. In future we would like to explore the possibility of retraction of feedbacks, i.e., in our model of reputation management it is equivalent to selling shares or exchanging shares. We believe such a model can be useful to encourage the sellers to remain consistent.

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